Radicales

- $$\frac{\sqrt{48}-\sqrt{75}}{\sqrt{25}+\sqrt{3}} = \frac{\sqrt{2^4\cdot 3}-\sqrt{5^2\cdot 3}}{\sqrt{5^2}+\sqrt{3}} = \frac{4\sqrt{3}-5\sqrt{3}}{5+\sqrt{3}} = \frac{-\sqrt{3}}{5+\sqrt{3}}$$

$$\frac{-\sqrt{3}}{(5+\sqrt{3})}\cdot\frac{(5-\sqrt{3})}{(5-\sqrt{3})} = \frac{-5\sqrt{3}+3}{25-3} = \boxed{\frac{-5\sqrt{3}+3}{22}}$$

- $$\sqrt{a^3}-2a\sqrt[4]{a^2}+3a\sqrt[6]{a^3}-\sqrt[8]{a^{12}} =$$

$$\sqrt{a^3}-2a\sqrt{a}+3a\sqrt{a}-\sqrt{a^3} =$$

$$\cancel{a\sqrt{a}}-2a\sqrt{a}+3a\sqrt{a}-\cancel{a\sqrt{a}} = \boxed{a\sqrt{a}}$$

- $$\frac{5}{\sqrt{6}}+\frac{2}{\sqrt{6}+3\sqrt{2}}-\frac{4\sqrt{2}}{\sqrt{3}} = \frac{5\sqrt{6}}{6}+\frac{2(\sqrt{6}-3\sqrt{2})}{6-9\cdot 2}-\frac{4\sqrt{6}}{3} =$$

$$= \frac{5\sqrt{6}}{6}-\frac{2\sqrt{6}-6\sqrt{2}}{12}-\frac{4\sqrt{6}}{3} = \frac{10\sqrt{6}-2\sqrt{6}+6\sqrt{2}-16\sqrt{6}}{12} =$$

$$= \frac{6\sqrt{2}-8\sqrt{6}}{12} = \boxed{\frac{3\sqrt{2}-4\sqrt{6}}{6}}$$

- $$\frac{\left(\sqrt[3]{a^2}\right)^4\cdot\left(a^2\cdot\sqrt{a}\right)^3}{\sqrt[6]{a^5}} = \frac{\sqrt[3]{a^8}\cdot a^6\cdot\sqrt{a^3}}{\sqrt[6]{a^5}} = \frac{a^3\cdot a^6\sqrt[3]{a^2}\cdot\sqrt{a}}{\sqrt[6]{a^5}} =$$

$$\frac{a^9\sqrt[6]{a^4\cdot a^3}}{\sqrt[6]{a^5}} = \frac{a^9\cdot a\sqrt[6]{a}}{\sqrt[6]{a^5}} = a^{10}\sqrt[6]{\frac{a}{a^5}} = a^{10}\cdot\frac{1}{\sqrt[6]{a^4}} =$$

$$\frac{a^{10}}{\sqrt[3]{a^2}}\cdot\frac{\sqrt[3]{a}}{\sqrt[3]{a}} = \frac{a^{10}\sqrt[3]{a}}{a} = \boxed{a^9\sqrt[3]{a}}$$

- $\left(\frac{\sqrt[6]{32}}{\sqrt{8}}\right)^{3/2} \cdot \left(\frac{1}{4}\right)^{-2} = \left(\frac{\sqrt[6]{2^5}}{\sqrt{2^3}}\right)^{3/2} \cdot 2^4 = \frac{\sqrt[6]{2^{15/2}}}{\sqrt{2^{9/2}}} \cdot 2^4 =$

$= \sqrt[6]{\frac{2^{15/2}}{2^{27/2}}} \cdot 2^4 = \frac{1}{\sqrt[6]{2^6}} \cdot 2^4 = 2^3 = \boxed{8}$

- $\left(-\frac{1}{27}\right)^{2/3} + \left(-\frac{1}{32}\right)^{2/5} = \sqrt[3]{\left(-\frac{1}{27}\right)^2} + \sqrt[5]{\left(-\frac{1}{32}\right)^2} =$

$\sqrt[3]{\frac{1}{3^6}} + \sqrt[5]{\frac{1}{2^{10}}} = \frac{1}{3^2} + \frac{1}{2^2} = \frac{1}{9} + \frac{1}{4} = \boxed{\frac{13}{36}}$

- $\sqrt{\frac{1}{2}} + 5\sqrt{2} + \sqrt[6]{8} - \sqrt{\frac{9}{8}} + \sqrt[4]{324} =$

$\sqrt{\frac{1}{2}} + 5\sqrt{2} + \sqrt{2} - \frac{3}{2}\sqrt{\frac{1}{2}} + \sqrt[4]{3^4 \cdot 2^2} = \sqrt{\frac{1}{2}} + 5\sqrt{2} + \sqrt{2} - \frac{3}{2}\sqrt{\frac{1}{2}} +$

$+ 3\sqrt{2} = \boxed{9\sqrt{2} - \frac{1}{2}\sqrt{\frac{1}{2}}}$

- $\left(\frac{\sqrt[3]{a^2} \cdot a^{5/2}}{\frac{1}{\sqrt{a^{-3}}}}\right)^6 = \frac{\sqrt[3]{a^{12}} \cdot a^{30/2}}{\frac{1}{\sqrt{a^{-18}}}} = \frac{a^4 \cdot a^{15}}{a^9} = \boxed{a^{10}}$

- $\left(\sqrt{2 \cdot \sqrt{\frac{1}{2}} \cdot \sqrt[3]{4}}\right) \cdot \sqrt[4]{2} = \left(\sqrt{\sqrt{2} \cdot \sqrt[3]{4}}\right) \cdot \sqrt[4]{2} =$

$\left(\sqrt{\sqrt[6]{2^3 \cdot 2^4}}\right) \cdot \sqrt[4]{2} = \sqrt[12]{2^7} \cdot \sqrt[4]{2} = \sqrt[12]{2^7 \cdot 2^3} = \sqrt[12]{2^{10}} = \underline{\underline{\sqrt[6]{2^5}}}$

- $\sqrt{75} + \frac{3\sqrt{18}}{3} - \frac{6}{\sqrt{12}} + \sqrt[4]{144} - |4-3\sqrt{2}| =$

$(4-3\sqrt{2}) < 0$

$\sqrt{3.5^2} + \sqrt{3^2.2} - \frac{6}{\sqrt{2^2.3}} + \sqrt[4]{2^4.3^2} - (-4+3\sqrt{2}) =$

$5\sqrt{3} + \cancel{3\sqrt{2}} - \frac{3}{\sqrt{3}} + 2\sqrt{3} + 4 - \cancel{3\sqrt{2}} = 4 + 7\sqrt{3} - \frac{3}{\sqrt{3}} =$

$= 4 + 7\sqrt{3} - \frac{\cancel{3}\sqrt{3}}{\cancel{3}} = \boxed{4+6\sqrt{3}}$

- $\sqrt[3]{1'08} - \sqrt[3]{2'56} + \sqrt[3]{1'215} - \sqrt[3]{0'625} =$

$\sqrt[3]{\frac{108}{100}} - \sqrt[3]{\frac{256}{100}} + \sqrt[3]{\frac{1215}{1000}} - \sqrt[3]{\frac{625}{1000}} = \sqrt[3]{\frac{27}{25}} - \sqrt[3]{\frac{64}{25}} +$

$+ \sqrt[3]{\frac{5.3^5}{2^3.5^3}} - \sqrt[3]{\frac{5^4}{2^3.5^3}} = 3\sqrt[3]{\frac{1}{25}} - 2^2\sqrt[3]{\frac{1}{25}} + \frac{3}{10}\sqrt[3]{3^2.5} -$

$- \frac{5}{10}\sqrt[3]{5} = \boxed{-\sqrt[3]{\frac{1}{25}} + \frac{3}{10}\sqrt[3]{45} - \frac{1}{2}\sqrt[3]{5}}$

- $\frac{\sqrt{72} + 3\sqrt{32} - \sqrt{8}}{\sqrt[3]{4}} = \frac{\sqrt{2^3.3^2} + 3\sqrt{2^5} - \sqrt{2^3}}{\sqrt[3]{2^2}} =$

$\frac{2.3\sqrt{2} + 3.2^2\sqrt{2} - 2\sqrt{2}}{\sqrt[3]{2^2}} = \frac{16\sqrt{2}}{\sqrt[3]{2^2}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} = \frac{16\sqrt{2}.\sqrt[3]{2}}{2} =$

$8\sqrt[6]{2^3.2^2} = \underline{8\sqrt[6]{2^5}}$

- $$\sqrt{\frac{(0'004)^4\cdot(0'0036)}{(120000)^2}} = \sqrt{\frac{(4.10^{-3})^4\cdot 3'6.10^{-3}}{(1'2.10^5)^2}} =$$

$$\sqrt{\frac{4^4.10^{-12}\cdot 3'6.10^{-3}}{1'2^2\cdot 10^{10}}} = \sqrt{\frac{2^8.3'6.10^{-15}}{1'2^2\cdot 10^{10}}} = \sqrt{640.10^{-25}} =$$

$$\sqrt{2^7.5.10^{-25}} = 2^3\sqrt{2.5.10^{-25}} = 2^3\sqrt{10^{-24}} = 2^3.10^{-12}$$

$$= \boxed{8.10^{-12}}$$

- $$\sqrt[3]{3a^2}\cdot\sqrt[4]{2ab}\cdot\sqrt[6]{a^3b^2} = \sqrt[12]{3^4a^8}\cdot\sqrt[12]{2^3a^3b^3}\cdot\sqrt[12]{a^6b^4} =$$

$$\sqrt[12]{3^4.2^3.a^8.a^3.a^6.b^3.b^4} = \sqrt[12]{3^4.2^3.a^{17}.b^7} = a\sqrt[12]{648a^5b^7}$$

- $$\frac{\sqrt[3]{5x}\cdot\sqrt[6]{8x^9}}{\sqrt{2x}\cdot\sqrt[3]{625x^4}} = \frac{\sqrt[3]{5x}\cdot\sqrt[6]{2^3x^9}}{\sqrt{2x}\cdot\sqrt[3]{5^4x^4}} = \frac{\sqrt[3]{5x}\cdot\sqrt{2x^3}}{\sqrt{2x}\cdot 5x\sqrt[3]{5x}} = \boxed{\frac{1}{5}}$$

- $$\sqrt{a\sqrt[5]{a}} : \sqrt{a\sqrt[4]{a}} = \sqrt{\sqrt[5]{a^6}} : \sqrt{\sqrt[4]{a^5}} = \sqrt[10]{a^6} : \sqrt[8]{a^5}$$

$$a^{6/10} : a^{5/8} = a^{6/10-\frac{5}{8}} = a^{-1/40} = \boxed{\frac{1}{\sqrt[40]{a}}}$$

- $\dfrac{\sqrt[3]{a}\cdot a^{1/4}\cdot a^{3/2}}{\sqrt[6]{a^5}} = \dfrac{\sqrt[3]{a}\cdot\sqrt[4]{a}\cdot\sqrt{a^3}}{\sqrt[6]{a^5}} =$

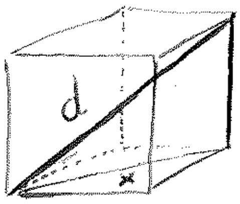
$$\sqrt[12]{\frac{a^4\cdot a^3\cdot a^{18}}{a^{10}}} = \sqrt[12]{\frac{a^{25}}{a^{10}}} = \sqrt[12]{a^{15}} = \boxed{a\sqrt[12]{a^3}}$$

- $\dfrac{2-\sqrt{2}}{2+\sqrt{2}} - \dfrac{2+\sqrt{2}}{2-\sqrt{2}} + \dfrac{4}{\sqrt[4]{64}} = \dfrac{(2-\sqrt{2})^2}{4-2} - \dfrac{(2+\sqrt{2})^2}{4-2} + \dfrac{4}{2\sqrt[4]{2^2}} =$

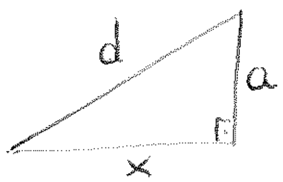
$$\frac{(2-\sqrt{2})^2}{2} - \frac{(2+\sqrt{2})^2}{2} + \frac{\cancel{4}\sqrt{2}}{\cancel{4}} = \frac{\cancel{4}-4\sqrt{2}+\cancel{2}-\cancel{4}-4\sqrt{2}-\cancel{2}+2\sqrt{2}}{2} =$$

$$\frac{-6\sqrt{2}}{2} = \boxed{-3\sqrt{2}}$$

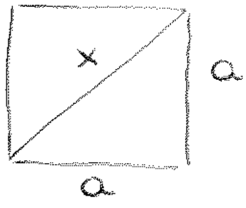
- Calcula la diagonal de un cubo de arista $a = \sqrt[3]{45}$ cm



$\Rightarrow$



(1) Calculo x (diagonal de una cara del cubo)



$x^2 = 2a^2 \Rightarrow \boxed{x = a\sqrt{2}}$

donde $a = \sqrt[3]{45}$ cm

(2) Calculo d con T° Pitágora

$$d^2 = (a\sqrt{2})^2 + a^2 \Rightarrow d^2 = a^2\cdot 2 + a^2$$

$$d^2 = 3a^2 \Rightarrow d = a\sqrt{3}$$

(3) Sustituyo a por su valor $\sqrt[3]{45}$

$$d = \sqrt[3]{45}\cdot\sqrt{3} = \sqrt[3]{3^2\cdot 5}\cdot\sqrt{3} = \sqrt[6]{3^4\cdot 5^2\cdot 3^3} = 3\sqrt[6]{3\cdot 5^2} = \underline{3\sqrt[6]{75}}$$

Logaritmos

- $\log_{216}\left(\frac{1}{\sqrt{6}}\right) = x$

$$216^{x} = \frac{1}{\sqrt{6}} \Rightarrow (2^3 \cdot 3^3)^x = \frac{1}{\sqrt{6}} \Rightarrow 6^{3x} = \frac{1}{\sqrt{6}}$$

$$6^{3x} = 6^{-1/2} \Rightarrow 3x = -\frac{1}{2} \Rightarrow \boxed{x = -\frac{1}{6}}$$

- $\log_{1/9}\left(\frac{\sqrt[4]{3}}{3}\right) = x$

$$\left(\frac{1}{9}\right)^{x} = \frac{\sqrt[4]{3}}{3} \Rightarrow \frac{1}{3^{2x}} = \frac{3^{1/4}}{3} \Rightarrow 3^{-2x} = 3^{1/4 - 1}$$

$$3^{-2x} = 3^{-3/4} \Rightarrow -2x = -\frac{3}{4} \Rightarrow \boxed{x = \frac{3}{8}}$$

- $\log_2 \frac{1}{\sqrt{2}} + \log_{81} \frac{1}{3} - \log_{\sqrt{5}} 625$

$$\log_2 \frac{1}{\sqrt{2}} = x \Rightarrow 2^x = \frac{1}{\sqrt{2}} \Rightarrow 2^x = 2^{-1/2} \Rightarrow x = -\frac{1}{2}$$

$$\log_{81} \frac{1}{3} = y \Rightarrow 81^y = \frac{1}{3} \Rightarrow 3^{4y} = 3^{-1} \Rightarrow y = -\frac{1}{4}$$

$$\log_{\sqrt{5}} 625 = z \Rightarrow (\sqrt{5})^z = 625 \Rightarrow 5^{z/2} = 5^4 \Rightarrow \frac{z}{2} = 4 \Rightarrow z = 8$$

$$-\frac{1}{2} - \frac{1}{4} - 8 = \frac{-2-1-32}{4} = \boxed{-\frac{35}{4}}$$

- $\log_4\left(3\sqrt[3]{128} + 5\sqrt[3]{2} + 3\sqrt[3]{54} - 5\sqrt[3]{16}\right) =$

$$\log_4\left(3\sqrt[3]{2^7} + 5\sqrt[3]{2} + 3\sqrt[3]{2 \cdot 3^3} - 5\sqrt[3]{2^4}\right) =$$

$$\log_4\left(3 \cdot 2^2 \sqrt[3]{2} + 5\sqrt[3]{2} + 3 \cdot 3\sqrt[3]{2} - 5 \cdot 2\sqrt[3]{2}\right) =$$

$$\log_4\left(16\sqrt[3]{2}\right) = \log_4\left(2^4 \cdot 2^{1/3}\right) = \log_4 2^{13/3} \Rightarrow 2^{2x} = 2^{13/3}$$

$$\boxed{x = 13/6}$$

- $\log_4\left(\sqrt{2\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}}\right) = \log_4 \sqrt[32]{2^{31}} \Rightarrow$

$\log_4 \sqrt[32]{2^{31}} = x \Rightarrow 2^{2x} = 2^{31/32} \Rightarrow 2x = \frac{31}{32} \quad \boxed{x = \frac{31}{64}}$

- $\log \dfrac{(0'64)^3 \cdot \sqrt[3]{0'32}}{80 \cdot \sqrt{6'25}} = \log \dfrac{\left(\frac{16}{25}\right)^3 \cdot \left(\frac{8}{25}\right)^{1/3}}{2^4 \cdot 5 \cdot \left(\frac{25}{4}\right)^{1/2}} =$

Sabiendo $\log 2 = 0'301$

$= \log \dfrac{\frac{2^{12}}{5^6} \cdot \frac{2}{5^{2/3}}}{2^4 \cdot 5 \cdot \frac{5}{2}} = \log \dfrac{2^{14}}{2^4 \cdot 5^{26/3}} = \log \dfrac{2^{10}}{5^{26/3}} =$

$\log 2^{10} - \log 5^{26/3} = 10 \cdot 0'301 - \frac{26}{3}\log 5 =$

$= \boxed{3'01 - \frac{26}{3}\log 5}$

- $\log_2 64 + \log_{\sqrt{2}} \frac{1}{4} - \log_3 9 - \log_2 \sqrt{2}$

$6 - 4 - 2 - \frac{1}{2} = \boxed{-\frac{1}{2}}$

- $\log_{\frac{1}{\sqrt{3}}}(81) = x \Rightarrow \left(\frac{1}{\sqrt{3}}\right)^x = 81 \Rightarrow 3^{-x/2} = 3^4$

$-\frac{x}{2} = 4 \Rightarrow \boxed{x = -8}$

- $\dfrac{\log \frac{1}{a} + \log\sqrt{a^3}}{\log a^4 - \log a}$ con $a \neq 1$

$\dfrac{\log\left(\frac{1}{a} \cdot \sqrt{a^3}\right)}{\log\left(\frac{a^4}{a}\right)} = \dfrac{\log a^{1/2}}{\log a^3} = \dfrac{\frac{1}{2}\cancel{\log a}}{3\cancel{\log a}} = \boxed{\frac{1}{6}}$

Intervalos

- $|x-3|>1$

$x-3>1 \qquad\qquad x-3<-1$

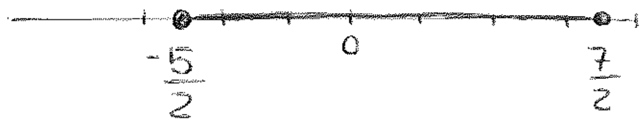
$x>4 \qquad\qquad x<2$

$$x \in (-\infty, 2) \cup (4, +\infty)$$

- $|2x-1| \leq 6$

$$-6 \leq (2x-1) \leq 6$$

$$-\frac{5}{2} \leq x \leq \frac{7}{2}$$



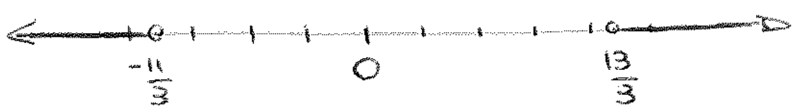
- $\left|\frac{1}{3}-x\right|>4$

$\frac{1}{3}-x>4 \qquad\qquad \frac{1}{3}-x<-4$

$\frac{1}{3}-4>x \qquad\qquad \frac{1}{3}+4<x$

$\frac{-11}{3}>x \qquad\qquad \frac{13}{3}<x$

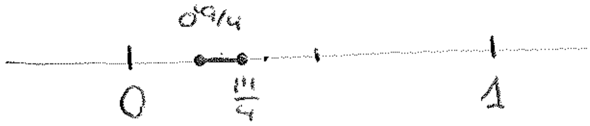
$$x \in (-\infty, -11/3) \cup (13/3, +\infty)$$



- $|4x-1| \leq 10^{-1}$

$|4x-1| \leq 0'1 \Rightarrow -0'1 \leq (4x-1) \leq 0'1$

$\frac{0'9}{4} \leq x \leq \frac{1'1}{4} \qquad x \in \left[\frac{0'9}{4}, \frac{1'1}{4}\right]$



- Calcula el valor de x en cada caso:

- $\log_x \left(\frac{1}{49}\right) = \frac{1}{4}$

$x^{1/4} = \frac{1}{49} \Rightarrow x^{1/4} = 7^{-2} \Rightarrow \boxed{x = 7^{-8}}$

- $x = \log_{1/9}\left(\frac{\sqrt[4]{3}}{3}\right) \Rightarrow \left(\frac{1}{9}\right)^x = \frac{\sqrt[4]{3}}{3} \Rightarrow$

$3^{-2x} = 3^{1/4-1} \Rightarrow 3^{-2x} = 3^{-3/4} \Rightarrow -2x = -\frac{3}{4} \Rightarrow$

$\Rightarrow \boxed{x = \frac{3}{8}}$

- $\log x = \frac{1}{2}(\log a + 3\log b) - \frac{1}{3}(\log c + 2\log b)$

$\log \frac{\sqrt{a} \cdot \sqrt{b^3}}{\sqrt[3]{c} \cdot \sqrt[3]{b^2}} = \log \frac{\sqrt{ab^3}}{\sqrt[3]{cb^2}} = \log \sqrt[6]{\frac{a^3 b^9}{c^2 \cancel{b^4}}} = \log \sqrt[6]{\frac{a^3 b^5}{c^2}}$