

Trigonometría

091

Resuelve las siguientes ecuaciones.

a) $\cos x \operatorname{tg} x = \frac{1}{2}$

f) $\operatorname{tg} x + \operatorname{sen} x = 0$

b) $\cos 2x + \operatorname{sen} 2x = 1$

g) $\operatorname{tg} x - \operatorname{sen} 2x = 0$

c) $\cos 2x - \operatorname{sen} 2x = 0$

h) $\frac{\operatorname{sen}(60^\circ - x)}{\cos x} = 1$

d) $\operatorname{sen} 2x + \cos x = 1$

i) $\operatorname{tg}\left(\frac{\pi}{4} - x\right) + \operatorname{tg} x - 1 = 0$

e) $\operatorname{sen} 2x + \operatorname{sen} 2x = 0$

j) $\operatorname{sen}(x + 30^\circ) + \cos(x + 60^\circ) = 1 + \cos 2x$

a) $\cos x \operatorname{tg} x = \frac{1}{2} \rightarrow \operatorname{sen} x = \frac{1}{2} \rightarrow \begin{cases} x_1 = 30^\circ + 360^\circ \cdot k \\ x_2 = 150^\circ + 360^\circ \cdot k \end{cases}$

b) $\cos 2x + \operatorname{sen} 2x = 1 \rightarrow \cos^2 x - \operatorname{sen}^2 x + 2 \operatorname{sen} x \cdot \cos x = \cos^2 x + \operatorname{sen}^2 x$
 $\rightarrow -2 \operatorname{sen}^2 x + 2 \operatorname{sen} x \cdot \cos x = 0 \rightarrow 2 \operatorname{sen} x (-\operatorname{sen} x + \cos x) = 0$

$\operatorname{sen} x = 0 \rightarrow \begin{cases} x_1 = 0^\circ + 360^\circ \cdot k \\ x_2 = 180^\circ + 360^\circ \cdot k \end{cases}$

$\operatorname{sen} x = \cos x \rightarrow \begin{cases} x_1 = 45^\circ + 360^\circ \cdot k \\ x_2 = 225^\circ + 360^\circ \cdot k \end{cases}$

c) $\cos 2x - \operatorname{sen} 2x = 0 \rightarrow \cos 2x = \operatorname{sen} 2x \rightarrow \begin{cases} x_1 = 22,5^\circ + 180^\circ \cdot k \\ x_2 = 112,5^\circ + 180^\circ \cdot k \end{cases}$

d) $\operatorname{sen} 2x + \cos x = (2 \operatorname{sen} x + 1) \cos x = 0 \rightarrow \begin{cases} x_1 = 90^\circ + 360^\circ \cdot k & x_3 = 210^\circ + k \cdot 360^\circ \\ x_2 = 270^\circ + 360^\circ \cdot k & x_4 = 330^\circ + k \cdot 360^\circ \end{cases}$

e) $\operatorname{sen} 2x + \operatorname{sen} 2x = 0 \rightarrow 2 \operatorname{sen} 2x = 0 \rightarrow \begin{cases} x_1 = 0^\circ + 180^\circ \cdot k \\ x_2 = 90^\circ + 180^\circ \cdot k \end{cases}$

f) $\operatorname{tg} x + \operatorname{sen} x = 0 \rightarrow \operatorname{sen} x \left(\frac{1}{\cos x} + 1 \right) = 0$

$\operatorname{sen} x = 0 \rightarrow \begin{cases} x_1 = 0^\circ + 360^\circ \cdot k \\ x_2 = 180^\circ + 360^\circ \cdot k \end{cases}$

$\frac{1}{\cos x} + 1 = 0 \rightarrow x_3 = 180^\circ + 360^\circ \cdot k$

g) $\operatorname{tg} x - \operatorname{sen} 2x = 0 \rightarrow \frac{\operatorname{sen} x}{\cos x} - 2 \operatorname{sen} x \cdot \cos x = 0 \rightarrow \operatorname{sen} x (1 - 2 \cos^2 x) = 0$

$\operatorname{sen} x = 0 \rightarrow \begin{cases} x_1 = 0^\circ + 360^\circ \cdot k \\ x_2 = 180^\circ + 360^\circ \cdot k \end{cases}$

$1 - 2 \cos^2 x = 0 \rightarrow \cos x = \sqrt{\frac{1}{2}} \rightarrow x_3 = 45^\circ + 360^\circ \cdot k$

h) $\frac{\operatorname{sen}(60^\circ - x)}{\cos x} = 1 \rightarrow \frac{\sqrt{3} \cos x - \operatorname{sen} x}{2 \cos x} = 1 \rightarrow \sqrt{3} - \operatorname{tg} x = 2$

$\rightarrow \operatorname{tg} x = -0,2679 \rightarrow x = 345^\circ + 360^\circ \cdot k$

$$\begin{aligned} \text{i) } \operatorname{tg}\left(\frac{\pi}{4} - x\right) + \operatorname{tg} x - 1 = 0 &\rightarrow \frac{1 - \operatorname{tg} x}{1 + \operatorname{tg} x} + \operatorname{tg} x - 1 = 0 \\ &\rightarrow \operatorname{tg}^2 x - \operatorname{tg} x = 0 \rightarrow \operatorname{tg} x(\operatorname{tg} x - 1) = 0 \end{aligned}$$

$$\operatorname{tg} x = 0 \rightarrow \begin{cases} x_1 = 0^\circ + 360^\circ \cdot k \\ x_2 = 180^\circ + 360^\circ \cdot k \end{cases}$$

$$\operatorname{tg} x - 1 = 0 \rightarrow \operatorname{tg} x = 1 \rightarrow \begin{cases} x_1 = 45^\circ + 360^\circ \cdot k \\ x_2 = 225^\circ + 360^\circ \cdot k \end{cases}$$

$$\text{j) } \operatorname{sen}(x + 30^\circ) + \operatorname{cos}(x + 60^\circ) = 1 + \operatorname{cos} 2x$$

$$\rightarrow \frac{\sqrt{3} \operatorname{sen} x}{2} + \frac{\operatorname{cos} x}{2} + \frac{\operatorname{cos} x}{2} - \frac{\sqrt{3} \operatorname{sen} x}{2} =$$

$$= \operatorname{cos}^2 x + \operatorname{sen}^2 x + \operatorname{cos}^2 x - \operatorname{sen}^2 x \rightarrow \operatorname{cos} x = 2 \operatorname{cos}^2 x \rightarrow \operatorname{cos} x(2 \operatorname{cos} x - 1) = 0$$

$$\operatorname{cos} x = 0 \rightarrow \begin{cases} x_1 = 90^\circ + 360^\circ \cdot k \\ x_2 = 270^\circ + 360^\circ \cdot k \end{cases}$$

$$2 \operatorname{cos} x - 1 = 0 \rightarrow \operatorname{cos} x = \frac{1}{2} \rightarrow \begin{cases} x_1 = 60^\circ + 360^\circ \cdot k \\ x_2 = 300^\circ + 360^\circ \cdot k \end{cases}$$

092

Resuelve estos sistemas de ecuaciones trigonométricas.

$$\begin{array}{l} \text{a) } \left. \begin{array}{l} \operatorname{sen}^2 x + \operatorname{sen}^2 y = 1 \\ \operatorname{cos}^2 x - \operatorname{cos}^2 y = \frac{1}{2} \end{array} \right\} \quad \text{b) } \left. \begin{array}{l} x + y = 120 \\ \operatorname{cos} x = \frac{1}{2 \operatorname{cos} y} + \operatorname{sen} x \cdot \operatorname{tg} y \end{array} \right\} \end{array}$$

$$\text{a) } \left. \begin{array}{l} \operatorname{sen}^2 x + \operatorname{sen}^2 y = 1 \\ \operatorname{cos}^2 x - \operatorname{cos}^2 y = \frac{1}{2} \end{array} \right\} \rightarrow \left. \begin{array}{l} \operatorname{sen}^2 x = 1 - \operatorname{sen}^2 y = \operatorname{cos}^2 y \\ \operatorname{cos}^2 x - \operatorname{sen}^2 y = \frac{1}{2} \end{array} \right\}$$

$$\operatorname{cos} 2x = \frac{1}{2} \rightarrow x = 30^\circ + 180^\circ \cdot k$$

$$\operatorname{cos}^2 y = \operatorname{sen}^2 30^\circ \rightarrow \operatorname{cos} y = \sqrt{\frac{1}{4}} \rightarrow y = 60^\circ + 180^\circ \cdot k$$

$$\begin{aligned} \text{b) } \left. \begin{array}{l} x + y = 120 \\ \operatorname{cos} x = \frac{1}{2 \operatorname{cos} y} + \operatorname{sen} x \cdot \operatorname{tg} y \end{array} \right\} &\xrightarrow{x = 120^\circ - y} \operatorname{cos}(120^\circ - y) = \\ &= \frac{1}{2 \operatorname{cos} y} + \operatorname{sen}(120^\circ - y) \operatorname{tg} y \end{aligned}$$

$$-\operatorname{cos}^2 y + \sqrt{3} \operatorname{sen} y \cdot \operatorname{cos} y = 1 + \sqrt{3} \operatorname{sen} y \cdot \operatorname{cos} y - \operatorname{sen}^2 y$$

$$\rightarrow \operatorname{cos}^2 y - \operatorname{sen}^2 y = 1 \rightarrow \operatorname{cos} 2y = -1 \rightarrow y = 90^\circ + 180^\circ \cdot k$$

$$x = 120^\circ - y = 120^\circ - 90^\circ - 180^\circ \cdot k = 30^\circ - 180^\circ \cdot k$$

Trigonometría

093

Resuelve las ecuaciones trigonométricas.

a) $4 \operatorname{sen} x - \operatorname{sec} x = 0$

b) $\frac{\cos^2 x}{2 \cos x + \operatorname{sen} x} = \operatorname{sen} x$

c) $\frac{1}{\cos x + \operatorname{sen} x} + 2 \operatorname{sen} x = 2 \cos x$

d) $\operatorname{sen} x (\operatorname{sen} x - 1) = 5 \cos^2 x - 4$

e) $2 \cos x - 1 = \operatorname{sec} x$

f) $2 \cos x + \operatorname{sen} x = 1$

g) $\operatorname{sen} x + \cos x = 0$

a) $4 \operatorname{sen} x - \operatorname{sec} x = 0 \rightarrow 4 \operatorname{sen} x \cdot \cos x - 1 = 0 \rightarrow 2 \operatorname{sen} 2x = 1 \rightarrow \operatorname{sen} 2x = \frac{1}{2}$
 $\rightarrow \begin{cases} x_1 = 15^\circ + 180^\circ \cdot k \\ x_2 = 75^\circ + 180^\circ \cdot k \end{cases}$

b) $\frac{\cos^2 x}{2 \cos x + \operatorname{sen} x} = \operatorname{sen} x \rightarrow \cos^2 x = 2 \cos x \cdot \operatorname{sen} x + \operatorname{sen}^2 x \rightarrow \cos 2x = \operatorname{sen} 2x$
 $\rightarrow \begin{cases} x_1 = 22,5^\circ + 180^\circ \cdot k \\ x_2 = 112,5^\circ + 180^\circ \cdot k \end{cases}$

c) $\frac{1}{\cos x + \operatorname{sen} x} + 2 \operatorname{sen} x = 2 \cos x \rightarrow \frac{2 \operatorname{sen} x \cdot \cos x + 2 \operatorname{sen}^2 x}{2 \cos^2 x + 2 \operatorname{sen} x \cdot \cos x} = 1$
 $\rightarrow \frac{\operatorname{sen} x (\cos x + \operatorname{sen} x)}{\cos x (\cos x + \operatorname{sen} x)} = 1 \rightarrow \operatorname{tg} x = 1 \rightarrow \begin{cases} x_1 = 45^\circ + 360^\circ \cdot k \\ x_2 = 225^\circ + 360^\circ \cdot k \end{cases}$

d) $\operatorname{sen} x (\operatorname{sen} x - 1) = 5 \cos^2 x - 4 \rightarrow \operatorname{sen}^2 x - \operatorname{sen} x = 5(1 - \operatorname{sen}^2 x) - 4$
 $6 \operatorname{sen}^2 x - \operatorname{sen} x - 1 = 0$
 $\rightarrow \operatorname{sen} x = -\frac{1}{3} \rightarrow \begin{cases} x_1 = 340^\circ 31' 44'' + 360^\circ \cdot k \\ x_2 = 199^\circ 28' 16'' + 360^\circ \cdot k \end{cases}$
 $\rightarrow \operatorname{sen} x = \frac{1}{2} \rightarrow \begin{cases} x_1 = 30^\circ + 360^\circ \cdot k \\ x_2 = 150^\circ + 360^\circ \cdot k \end{cases}$

e) $2 \cos x - 1 = \operatorname{sec} x \rightarrow 2 \cos^2 x - \cos x - 1 = 0$
 $\rightarrow \cos x = 1 \rightarrow \begin{cases} x_1 = 0^\circ + 360^\circ \cdot k \\ x_2 = 180^\circ + 360^\circ \cdot k \end{cases}$
 $\rightarrow \cos x = -\frac{1}{2} \rightarrow \begin{cases} x_1 = 120^\circ + 360^\circ \cdot k \\ x_2 = 240^\circ + 360^\circ \cdot k \end{cases}$

f) $2 \cos x + \operatorname{sen} x = 1 \rightarrow \sqrt{1 - \cos^2 x} = 1 - 2 \cos x \rightarrow 5 \cos^2 x - 4 \cos x = 0$
 $\rightarrow \cos x (5 \cos x - 4) = 0 \rightarrow \cos x = \frac{4}{5} \rightarrow \begin{cases} x_1 = 36^\circ 52' 11,6'' + 360^\circ \cdot k \\ x_2 = 323^\circ 7' 48,4'' + 360^\circ \cdot l \end{cases}$

g) $\operatorname{sen} x + \cos x = 0 \rightarrow \operatorname{sen} x = -\cos x \rightarrow \begin{cases} x_1 = 135^\circ + 360^\circ \cdot k \\ x_2 = 315^\circ + 360^\circ \cdot k \end{cases}$