2 REGLAS DE DERIVACIÓN

2.1 DERIVADA DE UNA POTENCIA, DE UNA SUMA Y DEL PRODUCTO POR UN NÚMERO

- $f(x) = k$ (constante) $\rightarrow$ $f'(x) = 0$
- $f(x) = x$ $\rightarrow$ $f'(x) = 1$
- $f(x) = x^n$ $\rightarrow$ $f'(x) = nx^{n-1}$
- $F(x) = f(x) \pm g(x)$ $\rightarrow$ $F'(x) = f'(x) \pm g'(x)$
- $F(x) = k \cdot f(x)$ $\rightarrow$ $F'(x) = k \cdot f'(x)$

EJERCICIO RESUELTO

Halla la derivada de las siguientes funciones:

a) $f(x) = x^4 - \frac{3}{4}x^3 + 2x - 1$ ***b)*** $f(x) = \sqrt{x}$

c) $f(x) = \frac{3}{5x^4}$ ***d)*** $f(x) = \frac{x^2}{\sqrt[3]{x}}$

RESOLUCIÓN

a) $f'(x) = 4x^3 - \frac{3}{4} \cdot 3x^2 + 2 = 4x^3 - \frac{9}{4}x^2 + 2$

b) $f(x) = x^{1/2} \rightarrow f'(x) = \frac{1}{2}x^{1/2-1} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

c) $f(x) = \frac{3}{5}x^{-4} \rightarrow f'(x) = \frac{3}{5} \cdot (-4) \cdot x^{-4-1} = \frac{-12}{5}x^{-5} = \frac{-12}{5x^5}$

d) $f(x) = \frac{x^2}{x^{1/3}} = x^{2-1/3} = x^{5/3} \rightarrow f'(x) = \frac{5}{3}x^{2/3} = \frac{5\sqrt[3]{x^2}}{3}$

Halla la derivada de cada una de estas funciones:

1. $f(x) = 2x + 1$ $\rightarrow$ $f'(x) =$
2. $f(x) = \frac{3x-2}{4}$ $\rightarrow$ $f'(x) =$
3. $f(x) = \frac{3}{4}$ $\rightarrow$ $f'(x) =$
4. $f(x) = \frac{x}{2} + 3$ $\rightarrow$ $f'(x) =$
5. $f(x) = x^3 - 3x^2 + 2$ $\rightarrow$ $f'(x) =$
6. $f(x) = \frac{3x^5}{5} - \frac{4x}{3} + 5$ $\rightarrow$ $f'(x) =$
7. $f(x) = \frac{4\pi - 2}{3}$ $\rightarrow$ $f'(x) =$

8 $f(x) = \frac{4}{3}(x^2 - \frac{3}{4}x + 2)$ $\rightarrow$ $f'(x) =$

9 $f(x) = \frac{x^2}{5} - \frac{x}{4} + \sqrt{5}$ $\rightarrow$ $f'(x) =$

10 $f(x) = \frac{x}{7} - \sqrt{7x} = \frac{x}{7} - \sqrt{7} \cdot \sqrt{x}$ $\rightarrow$ $f'(x) =$

11 $f(x) = \frac{1}{x}$ $\rightarrow$ $f'(x) =$

12 $f(x) = \frac{3}{x^2}$ $\rightarrow$ $f'(x) =$

13 $f(x) = \frac{5}{3x^3}$ $\rightarrow$ $f'(x) =$

14 $f(x) = \sqrt[3]{x^4}$ $\rightarrow$ $f'(x) =$

15 $f(x) = \frac{\sqrt{3x}}{x^2}$ $\rightarrow$ $f'(x) =$

16 $f(x) = \frac{3\sqrt{x^3}}{2x^4}$ $\rightarrow$ $f'(x) =$

17 $f(x) = \frac{2}{x} + \frac{x}{2}$ $\rightarrow$ $f'(x) =$

18 $f(x) = \frac{\sqrt[3]{x^2}}{3} - \frac{x}{3} + \sqrt{5}$ $\rightarrow$ $f'(x) =$

19 $f(x) = \sqrt[4]{\frac{1}{x^3}}$ $\rightarrow$ $f'(x) =$

20 $f(x) = \sqrt{\frac{3}{x^5}}$ $\rightarrow$ $f'(x) =$

21 $f(x) = \frac{2\sqrt{x}}{x} - \frac{3}{x^2} + \frac{1}{x}$ $\rightarrow$ $f'(x) =$

22 $f(x) = x - \frac{3\sqrt{5}}{4} + \frac{1}{x^2}$ $\rightarrow$ $f'(x) =$

23 $f(x) = \frac{x^2}{3} - \frac{3}{x^2} + \frac{3\sqrt{5}}{2}$ $\rightarrow$ $f'(x) =$

24 $f(x) = \frac{x^3}{3} - 4\sqrt{x} - \frac{2}{x^3} - x^2\sqrt{x}$ $\rightarrow$ $f'(x) =$

25 $f(x) = \frac{x^2 - 3x + 1}{x} = \frac{x^2}{x} - \frac{3x}{x} + \frac{1}{x}$ $\rightarrow$ $f'(x) =$

2.2 OTRAS REGLAS DE DERIVACIÓN

- $F(x) = f(x) \cdot g(x) \rightarrow F'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$
- $F(x) = \dfrac{f(x)}{g(x)} \rightarrow F'(x) = \dfrac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}$
- $f(x) = sen\ x \rightarrow f'(x) = cos\ x$
- $f(x) = cos\ x \rightarrow f'(x) = -sen\ x$
- $f(x) = tg\ x \rightarrow f'(x) = 1 + tg^2\ x = \dfrac{1}{cos^2\ x}$
- $f(x) = arc\,sen\ x \rightarrow f'(x) = \dfrac{1}{\sqrt{1 - x^2}}$
- $f(x) = arc\,cos\ x \rightarrow f'(x) = \dfrac{-1}{\sqrt{1 - x^2}}$
- $f(x) = arc\,tg\ x \rightarrow f'(x) = \dfrac{1}{1 + x^2}$
- $f(x) = e^x \rightarrow f'(x) = e^x$
- $f(x) = a^x \rightarrow f'(x) = a^x \cdot ln\ a$
- $f(x) = ln\ x \rightarrow f'(x) = \dfrac{1}{x}$
- $f(x) = log_a\ x \rightarrow f'(x) = \dfrac{1}{x} \cdot \dfrac{1}{ln\ a}$

Halla la derivada de las siguientes funciones:

1. $f(x) = 3\ sen\ x - 2\ cos\ x \rightarrow f'(x) =$
2. $f(x) = 4\ tg\ x + e^x \rightarrow f'(x) =$
3. $f(x) = \underbrace{x}_{F(x)} \cdot \underbrace{ln\ x}_{G(x)} \rightarrow f'(x) = \underbrace{1}_{F'(x)} \cdot \underbrace{ln\ x}_{G(x)} + \underbrace{x}_{F(x)} \cdot \underbrace{\dfrac{1}{x}}_{G'(x)} = ln\ x + 1$
4. $f(x) = x\ e^x \rightarrow f'(x) =$
5. $f(x) = (x^2 + 1) \cdot sen\ x \rightarrow f'(x) =$
6. $f(x) = 2^x \cdot tg\ x \rightarrow f'(x) =$
7. $f(x) = (x^2 - \dfrac{x}{3}\ e^x) \rightarrow f'(x) =$
8. $f(x) = (x^3 - 2x + 1) \cdot cos\ x \rightarrow f'(x) =$

9. $f(x) = 3^x + \ln x - \frac{1}{x} \rightarrow f'(x) =$

10. $f(x) = 2^x + \log_2 x \rightarrow f'(x) =$

11. $f(x) = x^2 e^x + 2x \ln x \rightarrow f'(x) =$

12. $f(x) = \sqrt{x}\, \text{sen}\, x - \log_3 5 \rightarrow f'(x) =$

13. $f(x) = \frac{4x}{x+1} = \frac{F(x)}{G(x)} \rightarrow f'(x) = \frac{\overbrace{F'(x) \cdot G(x) - F(x) \cdot G'(x)}}{\underbrace{(G(x))^2}} = \frac{\overbrace{4} \cdot \overbrace{(x+1)} - \overbrace{4x} \cdot \overbrace{1}}{(x+1)^2} = \frac{4x + 4 - 4x}{(x+1)^2} = \frac{4}{(x+1)^2}$

14. $f(x) = \frac{x^2 - 1}{2x + 2} \rightarrow f'(x) =$

15. $f(x) = \frac{x+1}{x-2} \rightarrow f'(x) =$

16. $f(x) = \frac{\ln x}{x} \rightarrow f'(x) =$

17. $f(x) = \frac{e^x + e^{-x}}{2} \rightarrow f'(x) =$

18. $f(x) = \frac{1}{x^2 + 1} \rightarrow f'(x) =$

19. $f(x) = \frac{x^3}{x+2} \rightarrow f'(x) =$

20. $f(x) = \frac{2x - 1}{3x + 2} \rightarrow f'(x) =$

21. $f(x) = \frac{x^2}{x^2 - 1} \rightarrow f'(x) =$

22. $f(x) = \frac{\sqrt{x}}{x+2} \rightarrow f'(x) =$

23. $f(x) = (x^2 - 1)\sqrt{x} \rightarrow f'(x) =$

24. $f(x) = 3\, \text{arc sen}\, x \rightarrow f'(x) =$

25. $f(x) = 2\, \text{arc cos}\, x + e^x \rightarrow f'(x) =$

26. $f(x) = 5\, \text{arc tg}\, x \rightarrow f'(x) =$

27. $f(x) = \frac{x e^x - \ln x}{2} \rightarrow f'(x) =$

28. $f(x) = 3^x\, \text{sen}\, x - \log_2 x \rightarrow f'(x) =$

2.3 DERIVADA DE UNA FUNCIÓN COMPUESTA. REGLA DE LA CADENA

$$F(x) = (g\ f)(x) = g(f(x)) \quad \rightarrow \quad F'(x) = g'(f(x)) \cdot f'(x)$$

Las reglas de derivación aplicadas a funciones compuestas quedan así:

- $F(x) = (f(x))^n \rightarrow F'(x) = n \cdot (f(x))^{n-1} \cdot f'(x)$
- $F(x) = sen\,(f(x)) \rightarrow F'(x) = cos\,(f(x)) \cdot f'(x)$
- $F(x) = cos\,(f(x)) \rightarrow F'(x) = -sen\,(f(x)) \cdot f'(x)$
- $F(x) = tg\,(f(x)) \rightarrow F'(x) = (1 + tg^2\,(f(x)) \cdot f'(x) = \dfrac{f'(x)}{cos^2\,(f(x))}$
- $F(x) = arc\,sen\,(f(x)) \rightarrow F'(x) = \dfrac{f'(x)}{\sqrt{1 - (f(x))^2}}$
- $F(x) = arc\,cos\,(f(x)) \rightarrow F'(x) = \dfrac{-f'(x)}{\sqrt{1 - (f(x))^2}}$
- $F(x) = arc\,tg\,(f(x)) \rightarrow F'(x) = \dfrac{f'(x)}{1 + (f(x))^2}$
- $F(x) = e^{f(x)} \rightarrow F'(x) = e^{f(x)} \cdot f'(x)$
- $F(x) = a^{f(x)} \rightarrow F'(x) = a^{f(x)} \cdot f'(x) \cdot ln\,a$
- $F(x) = ln\,(f(x)) \rightarrow F'(x) = \dfrac{f'(x)}{f(x)}$
- $F(x) = log_a\,(f(x)) \rightarrow F'(x) = \dfrac{f'(x)}{f(x)} \cdot \dfrac{1}{ln\,a}$

EJERCICIO RESUELTO

Halla la derivada de estas funciones:

a) $f(x) = (x^2 - 3x)^7$

b) $g(x) = sen^3\,x = (sen\,x)^3$

c) $h(x) = ln^3\,(x^2 + 3) = [ln\,(x^2 + 3)]^3$

RESOLUCIÓN

a) $f'(x) = 7(x^2 - 3x)^6 \cdot (2x - 3) = (14x - 21)(x^2 - 3x)^6$

b) $g'(x) = 3\,sen^2\,x \cdot cos\,x$

c) $h'(x) = 3\,[ln\,(x^2 + 3)]^2 \cdot \dfrac{1}{x^2 + 3} \cdot 2x = \dfrac{6x\,ln^2\,(x^2 + 3)}{x^2 + 3}$

Halla la derivada de las siguientes funciones:

1. $f(x) = (x^2 + 5)^6 \rightarrow f'(x) =$
2. $f(x) = sen\,(x^2 - 1) \rightarrow f'(x) =$
3. $f(x) = cos\,(ln\,x) \rightarrow f'(x) =$
4. $f(x) = tg\,(2x - 3x^2) \rightarrow f'(x) =$
5. $f(x) = e^{3x^2 + 1} \rightarrow f'(x) =$
6. $f(x) = 2^{4x + 1} \rightarrow f'(x) =$
7. $f(x) = cos^2 x \rightarrow f'(x) =$
8. $f(x) = e^{3x} \rightarrow f'(x) =$
9. $f(x) = ln\,(3x^2 - 6) \rightarrow f'(x) =$
10. $f(x) = ln\left(\frac{3x^2 - 1}{2}\right) \rightarrow f'(x) =$
11. $f(x) = arc\,tg\,(3x^2 + 2x) \rightarrow f'(x) =$
12. $f(x) = arc\,sen\,(x^2) \rightarrow f'(x) =$
13. $f(x) = arc\,cos\,(x^3 - 1) \rightarrow f'(x) =$
14. $f(x) = sen\,(3x^2 - 1)^2 \rightarrow f'(x) =$
15. $f(x) = sen^2\,(3x^2 - 1) \rightarrow f'(x) =$
16. $f(x) = 3^{cos\,x} \rightarrow f'(x) =$
17. $f(x) = ln\left(\frac{x + 1}{x - 2}\right) \rightarrow f'(x) =$
18. $f(x) = \left(\frac{x^2 - 1}{x + 2}\right)^2 \rightarrow f'(x) =$
19. $f(x) = \sqrt{x^2 - 4x} \rightarrow f'(x) =$
20. $f(x) = \frac{x + 1}{(x - 2)^2} \rightarrow f'(x) =$
21. $f(x) = \frac{(2x + 1)^2}{x - 1} \rightarrow f'(x) =$
22. $f(x) = \frac{(3x - 1)^2}{2x + 1} \rightarrow f'(x) =$
23. $f(x) = \frac{e^x}{(x - 1)^2} \rightarrow f'(x) =$

2.4 EJERCICIOS DE RECAPITULACIÓN

Halla la derivada de las siguientes funciones:

1. $f(x) = \frac{x^3}{3} - \frac{x^2}{4} + \frac{2}{3} \rightarrow f'(x) =$
2. $f(x) = \frac{x^5}{3} - \frac{2}{x^2} + 3 \rightarrow f'(x) =$
3. $f(x) = \frac{x^2 - 2x + 1}{5} \rightarrow f'(x) =$
4. $f(x) = (3x - 2)\, e^x \rightarrow f'(x) =$
5. $f(x) = \sqrt{x} - \frac{2}{x^3} + \sqrt{5} \rightarrow f'(x) =$
6. $f(x) = \frac{1}{x} - \frac{\sqrt[3]{x}}{3} + 2x^2 \rightarrow f'(x) =$
7. $f(x) = \frac{\sqrt[3]{x}}{x^2} - \frac{x^2 - 1}{3} \rightarrow f'(x) =$
8. $f(x) = \frac{x^3 - 3x^4 + 2x + 1}{x} \rightarrow f'(x) =$
9. $f(x) = \frac{3}{2x^2} - \frac{2x^2}{3} + ln\, 5 \rightarrow f'(x) =$
10. $f(x) = \sqrt{\frac{2}{x^3}} - \frac{x^2}{3} + \sqrt{2} \rightarrow f'(x) =$
11. $f(x) = \frac{2\sqrt{3}}{4} + \frac{3\, ln\, x}{2} \rightarrow f'(x) =$
12. $f(x) = sen\, x \cdot cos\, x \rightarrow f'(x) =$
13. $f(x) = \frac{e^x}{x^2 - 1} \rightarrow f'(x) =$
14. $f(x) = \frac{x^2 - 1}{2x + 1} \rightarrow f'(x) =$
15. $f(x) = (x^2 - 1)\, e^x - ln\, x \rightarrow f'(x) =$
16. $f(x) = 2^x - 3\, tg\, x \rightarrow f'(x) =$
17. $f(x) = x^3\, e^x + x^2\, sen\, x \rightarrow f'(x) =$
18. $f(x) = \frac{x - 1}{3x - 2} \rightarrow f'(x) =$
19. $f(x) = \frac{\sqrt{x}}{sen\, x} \rightarrow f'(x) =$

20 $f(x) = (x^2 - 1)^4 \rightarrow f'(x) =$

21 $f(x) = \left(\frac{x-1}{x+2}\right)^3 \rightarrow f'(x) =$

22 $f(x) = \frac{2x-1}{(x+1)^2} \rightarrow f'(x) =$

23 $f(x) = \frac{x+1}{(x-1)^3} \rightarrow f'(x) =$

24 $f(x) = ln\left(\frac{x-1}{x+4}\right) \rightarrow f'(x) =$

25 $f(x) = cos^2(3x - 2) \rightarrow f'(x) =$

26 $f(x) = \sqrt{sen\ x} \rightarrow f'(x) =$

27 $f(x) = ln(sen\ x^2) \rightarrow f'(x) =$

28 $f(x) = e^{4x-1} \cdot sen(3x^2) \rightarrow f'(x) =$

29 $f(x) = 2^{4x^2-1} \cdot ln(8x) \rightarrow f'(x) =$

30 $f(x) = \frac{(2x+3)^2}{1-x} \rightarrow f'(x) =$

31 $f(x) = tg\left(\frac{2}{x-3}\right) \rightarrow f'(x) =$

32 $f(x) = \frac{e^{5x+1}}{x+2} \rightarrow f'(x) =$

33 $f(x) = \frac{ln^2 x}{x} \rightarrow f'(x) =$

34 $f(x) = \frac{x\,e^x}{x+2} \rightarrow f'(x) =$

35 $f(x) = \frac{\sqrt{x-1}}{3x+4} \rightarrow f'(x) =$

36 $f(x) = \sqrt{\frac{3x+1}{x+2}} \rightarrow f'(x) =$

37 $f(x) = arc\,tg(x^2 + 2) \rightarrow f'(x) =$

38 $f(x) = \sqrt{arc\,tg\ x} \rightarrow f'(x) =$

39 $f(x) = \frac{3\ arc\,sen(2x-1)}{4} \rightarrow f'(x) =$

40 $f(x) = arc\,cos(\sqrt{x}) \rightarrow f'(x) =$

b) $f'(2) = 0$ (la recta tangente es horizontal, luego tiene pendiente 0).

c) $f'(4)$ es la pendiente de la recta que pasa por (3, 3) y (4, 0):

$f'(4) = \frac{0-3}{4-3} = \frac{-3}{1} = -3$

9 $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{(a+h)^2 - a^2}{h} =$

$= \lim_{h \to 0} \frac{a^2 + h^2 + 2ah - a^2}{h} = \frac{h^2 + 2ah}{h} =$

$= \lim_{h \to 0} \frac{h(h + 2a)}{h} = \lim_{h \to 0} (h + 2a) = 2a$

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10 a) $f'(x) = 3$

b) $f'(x) = \frac{1}{5}$

c) $f'(x) = \frac{-1}{2}$

d) $f'(x) = 1$

e) $f'(x) = 2x$

f) $f'(x) = 0$

g) $f'(x) = 0$

h) $f'(x) = 4x - 1$

i) $f'(x) = \frac{-1}{x^2}$

j) $f'(x) = \frac{-1}{(x+1)^2}$

k) $f'(x) = \frac{-6}{(3x-1)^2}$

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1 $f'(x) = 2$

2 $f'(x) = \frac{3}{4}$

3 $f'(x) = 0$

4 $f'(x) = \frac{1}{2}$

5 $f'(x) = 3x^2 - 6x$

6 $f'(x) = 3x^4 - \frac{4}{3}$

7 $f'(x) = 0$

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8 $f'(x) = \frac{4}{3}\left(2x - \frac{4}{3}\right)$

9 $f'(x) = \frac{2x}{5} - \frac{1}{4}$

10 $f'(x) = \frac{1}{7} - \frac{\sqrt{7}}{2\sqrt{x}}$

11 $f'(x) = \frac{-1}{x^2}$

12 $f'(x) = \frac{-6}{x^3}$

13 $f'(x) = \frac{-5}{x^4}$

14 $f'(x) = \frac{4}{3}\sqrt[3]{x}$

15 $f'(x) = \sqrt{3} \cdot x^{-3/2} \rightarrow f'(x) = \frac{-3\sqrt{3}}{2} \cdot \frac{1}{\sqrt{x^5}} =$

$= \frac{-3\sqrt{3}}{2\sqrt{x^5}}$

16 $f'(x) = \frac{3}{2}x^{-5/2} \rightarrow f'(x) = \frac{-15}{4\sqrt{x^7}}$

17 $f'(x) = \frac{-2}{x^2} + \frac{1}{2}$

18 $f'(x) = \frac{1}{3\sqrt[3]{x}} - \frac{1}{3}$

19 $f'(x) = x^{-3/4} \rightarrow f'(x) = \frac{-3}{4\sqrt[4]{x^7}}$

20 $f'(x) = \frac{\sqrt{3}}{\sqrt{x^5}} = \sqrt{3} \cdot x^{-5/2} \rightarrow f'(x) = \frac{-5\sqrt{3}}{2\sqrt{x^7}}$

21 $f'(x) = 2x^{-1/2} - 3x^{-2} + x^{-1} \rightarrow$

$\rightarrow f'(x) = \frac{-1}{\sqrt{x^3}} + \frac{6}{x^3} - \frac{1}{x^2}$

22 $f'(x) = 1 - \frac{2}{x^3}$

23 $f'(x) = \frac{2x}{3} + \frac{6}{x^3}$

24 $f'(x) = \frac{x^3}{3} - 4x^{1/2} + 2x^{-3} + x^{2/5} \rightarrow$

$\rightarrow f'(x) = x^2 - \frac{2}{\sqrt{x}} - \frac{6}{x^4} + \frac{5\sqrt{x^3}}{2}$

25 $f'(x) = 1 - \frac{1}{x^2}$

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1 $f'(x) = 3 \cos x + 2 \operatorname{sen} x$

2. $f'(x) = 4(1 + tg^2 x) + e^x = 4 + 4\,tg^2 x + e^x$

4. $f'(x) = 1 \cdot e^x + x \cdot e^x = e^x + xe^x$

5. $f'(x) = 2x \cdot sen\,x + (x^2 + 1) \cdot \cos x =$
$= 2x\,sen\,x + x^2\,cos\,x + cos\,x$

6. $f'(x) = 2^x \cdot ln\,2 \cdot tg\,x + 2^x \cdot (1 + tg^2 x)$

7. $f'(x) = 2x - \left(\frac{1}{3} \cdot e^x + \frac{x}{3} \cdot e^x\right) =$
$= 2x - \frac{1}{3} \cdot e^x - \frac{x}{3} \cdot e^x$.

8. $f'(x) = (3x^2 - 2) \cdot cos\,x + (x^3 - 2x + 1) \cdot (-sen\,x) =$
$= (3x^2 - 2) \cdot cos\,x - (x^3 - 2x + 1) \cdot sen\,x$

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9. $f'(x) = 3^x \cdot ln\,3 + \frac{1}{x} - \frac{1}{x^2}$

10. $f'(x) = 2^x \cdot ln\,2 + \frac{1}{x} - \frac{1}{ln\,2}$

11. $f'(x) = 2x \cdot e^x + x^2 \cdot e^x + 2 \cdot ln\,x + 2x \cdot \frac{1}{x} =$
$= 2xe^x + x^2 e^x + 2\,ln\,x + 2$

12. $f'(x) = \frac{1}{2\sqrt{x}}\,sen\,x + \sqrt{x}\,\cos x =$
$= \frac{sen\,x}{2\sqrt{x}} + \sqrt{x}\,\cos x = \frac{sen\,x + 2x\,cos\,x}{2\sqrt{x}}$

13. $f(x) = \frac{(x+1) \cdot (x-1)}{2(x+1)} = \frac{(x-1)}{2} \rightarrow$
$\rightarrow f'(x) = \frac{1}{2}$

Otra forma:

$f'(x) = \frac{2x(2x+2) - (x^2-1) \cdot 2}{(2x+2)^2} =$

$= \frac{2x^2 + 4x + 2}{(2x+2)^2} = \frac{2(x+1)^2}{4(x+1)^2} = \frac{1}{2}$

14. $f'(x) = \frac{-3}{(x-2)^2}$

15. $f'(x) = \frac{\frac{1}{x} \cdot x - ln\,x \cdot 1}{x^2} = \frac{1 - ln\,x}{x^2}$

16. $f'(x) = \frac{e^x - e^{-x}}{2}$

17. $f'(x) = \frac{-2x}{(x^2+1)^2}$

18. $f'(x) = \frac{3x^2(x+2) - x^3 \cdot 1}{(x+2)^2} = \frac{2x^3 + 6x^2}{(x+2)^2}$

19. $f'(x) = \frac{2(3x+2) - (2x-1) \cdot 3}{(3x+2)^2} = \frac{7}{(3x+2)^2}$

20. $f'(x) = \frac{2x(x^2-1) - x^2 \cdot 2x}{(x^2-1)^2} = \frac{-2x}{(x^2-1)^2}$

21. $f'(x) = \frac{\frac{1}{2\sqrt{x}}(x+2) - \sqrt{x} \cdot 1}{(x+2)^2} = \frac{-x+2}{2\sqrt{x} \cdot (x+2)^2}$

22. $f'(x) = 2x \cdot \sqrt{x} + (x^2 - 1) \cdot \frac{1}{2\sqrt{x}} =$
$= \frac{4x^2 + x^2 - 1}{2\sqrt{x}} = \frac{5x^2 - 1}{2\sqrt{x}}$

23. $f'(x) = \frac{3}{\sqrt{1 - x^2}}$

24. $f'(x) = \frac{-2}{\sqrt{1 - x^2}} + e^x$

25. $f'(x) = \frac{5}{1 + x^2}$

26. $f'(x) = \frac{1 \cdot e^x + x\,e^x - \frac{1}{x}}{2} = \frac{x\,e^x + x^2\,e^x - 1}{2x}$

27. $f'(x) = 3^x \cdot ln\,3 \cdot sen\,x + 3^x \cdot cos\,x - \left(log_2 x + \right.$
$\left. + x \cdot \frac{1}{x} \cdot \frac{1}{ln\,2}\right) = 3^x\,ln\,3 \cdot sen\,x +$
$+ 3^x\,cos\,x - log_2 x - \frac{1}{ln\,2}$

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1. $f'(x) = 6(x^2 + 5)^5 (2x) = 12x(x^2 + 5)^5$

2. $f'(x) = cos(x^2 - 1) \cdot 2x = 2x\,cos(x^2 - 1)$

3. $f'(x) = -sen(ln\,x) \cdot \frac{1}{x} = \frac{-1}{x}\,sen(ln\,x)$

4. $f'(x) = [1 + tg^2(2x - 3x^2)] \cdot (2 - 6x)$

5. $f'(x) = e^{3x^2 + 1} \cdot 6x = 6x \cdot e^{3x^2 + 1}$

6. $f'(x) = 2^{4x+1} \cdot ln\,2 \cdot 4 = (4 \cdot ln\,2) \cdot 2^{4x+1}$

7. $f'(x) = 2\,cos\,x\,(-sen\,x) = -2\,cos\,x\,\,sen\,x$

8. $f'(x) = e^{3x} \cdot 3 = 3 \cdot e^{3x}$

9. $f'(x) = \frac{6x}{3x^2 - 6}$

10. $f'(x) = \dfrac{3x}{\dfrac{3x^2-1}{2}} = \dfrac{6x}{3x^2-1}$

11. $f'(x) = \dfrac{6x+2}{1+(3x^2+2x)^2} = \dfrac{6x+2}{9x^4+12x^3+4x^2+1}$

12. $f'(x) = \dfrac{2x}{\sqrt{1-x^4}}$

13. $f'(x) = \dfrac{-3x^2}{\sqrt{1-(x^3-1)^2}} = \dfrac{-3x^2}{\sqrt{2x^3-x^6}} =$

$= \dfrac{-3x^2}{x\sqrt{2x-x^4}} = \dfrac{-3x}{\sqrt{2x-x^4}}$

14. $f'(x) = (\cos(3x^2-1)^2) \cdot 2(3x^2-1) \cdot 6x =$

$= 12x(3x^2-1)\cos(3x^2-1)^2$

15. $f'(x) = 2\,sen(3x^2-1) \cdot \cos(3x^2-1) \cdot 6x =$

$= 12x\,sen(3x^2-1)\cos(3x^2-1)$

16. $f'(x) = 3^{\cos x} \cdot ln\,3 \cdot (-sen\,x) = -sen\,x \cdot ln\,3 \cdot 3^{\cos x}$

17. $f'(x) = \dfrac{1}{\dfrac{x+1}{x-2}} \cdot \dfrac{1\cdot(x-2)-(x+1)\cdot 1}{(x-2)^2} =$

$= \dfrac{(x-2)}{(x+1)} \cdot \dfrac{-3}{(x-2)^2} = \dfrac{-3}{(x+1)(x-2)} =$

$= \dfrac{-3}{x^2-x-2}$

18. $f'(x) = 2\left(\dfrac{x^2-1}{x+2}\right) \cdot \dfrac{2x(x+2)-(x^2-1)\cdot 1}{(x+2)^2} =$

$= \dfrac{2(x^2-1)(x^2+4x+1)}{(x+2)^3}$

19. $f'(x) = \dfrac{2x-4}{2\sqrt{x^2-4x}} = \dfrac{x-2}{\sqrt{x^2-4x}}$

20. $f'(x) = \dfrac{1\cdot(x-2)^2-(x+1)\cdot 2(x-2)}{(x-2)^4} =$

$= \dfrac{(x-2)[1-2(x+1)]}{(x-2)^4} = \dfrac{2x-1}{(x-2)^3}$

21. $f'(x) = \dfrac{2(2x+1)\cdot 2(x-1)-(2x+1)^2\cdot 1}{(x-1)^2} =$

$= \dfrac{(2x+1)(2x-5)}{(x-1)^2} = \dfrac{4x^2-8x-5}{(x-1)^2}$

22. $f'(x) = \dfrac{2(3x-1)\cdot 3(2x+1)-(3x-1)^2\cdot 2}{(2x+1)^2} =$

$= \dfrac{(3x-1)(6x+8)}{(2x+1)^2} = \dfrac{18x^2+18x-8}{(2x+1)^2}$

23. $f'(x) = \dfrac{e^x(x-1)^2-e^x\cdot 2(x-1)}{(x-1)^4} =$

$= \dfrac{e^x(x-1)(x-1-2)}{(x-1)^4} = \dfrac{e^x(x-3)}{(x-1)^3}$

Página 14

1. $f'(x) = x^2 - \dfrac{x}{2}$

2. $f'(x) = \dfrac{5x^4}{3} + \dfrac{4}{x^3}$

3. $f'(x) = \dfrac{2x-2}{5}$

4. $f'(x) = 3e^x + (3x-2)e^x = (3x+1)e^x$

5. $f'(x) = \dfrac{1}{2\sqrt{x}} + \dfrac{6}{x^4}$

6. $f'(x) = \dfrac{-1}{x^2} - \dfrac{1}{9\sqrt[3]{x^4}} + 4x$

7. $f'(x) = \dfrac{-5}{3\sqrt[3]{x^8}} - \dfrac{2x}{3}$

8. $f'(x) = x^2 - 3x^3 + 2 + \dfrac{1}{x}$

$f'(x) = 2x - 9x^2 - \dfrac{1}{x^2}$

9. $f'(x) = \dfrac{-3}{x^3} - \dfrac{4x}{3}$

10. $f'(x) = \dfrac{-3\sqrt{2}}{2\sqrt{x^5}} - \dfrac{2x}{3}$

11. $f'(x) = \dfrac{3}{2x}$

12. $f'(x) = \cos^2 x - sen^2 x$

13. $f'(x) = \dfrac{e^x(x^2-1)-e^x\cdot 2x}{(x^2-1)^2} = \dfrac{(x^2-2x-1)e^x}{(x^2-1)^2}$

14. $f'(x) = \dfrac{2x(2x+1)-(x^2-1)\cdot 2}{(2x+1)^2} = \dfrac{2x^2+2x+2}{(2x+1)^2}$

15. $f'(x) = 2x\,e^x + (x^2-1)e^x - \dfrac{1}{x} =$

$= (x^2+2x-1)e^x - \dfrac{1}{x}$

16. $f'(x) = 2^x \cdot ln\,2 - 3(1+tg^2 x) =$

$= 2^x \cdot ln\,2 - 3 - 3\,tg^2 x$

17. $f'(x) = 3x^2 e^x + x^3 e^x + 2x\,sen\,x + x^2 \cos x$

18. $f'(x) = \dfrac{1\cdot(3x-2)-(x-1)\cdot 3}{(3x-2)^2} = \dfrac{1}{(3x-2)^2}$

$f'(x) = \dfrac{\frac{1}{2\sqrt{x}}\ sen\ x - \sqrt{x}\ cos\ x}{sen^2\ x} =$

$= \dfrac{sen\ x - 2x\ cos\ x}{2\sqrt{x}\ sen^2\ x}$

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$f'(x) = 4\ (x^2 - 1)^3 \cdot 2x = 8x\ (x^2 - 1)^3$

$f'(x) = 3\left(\dfrac{x-1}{x+2}\right)^2 \cdot \dfrac{3}{(x+2)^2} = \dfrac{9\ (x-1)^2}{(x+2)^4}$

$f'(x) = \dfrac{2\ (x+1)^2 - (2x-1) \cdot 2\ (x+1)}{(x+1)^4} =$

$= \dfrac{(x+1) \cdot (2x + 2 - 4x + 2)}{(x+1)^4} = \dfrac{-2x+4}{(x+1)^3}$

$f'(x) = \dfrac{(x-1)^3 - (x+1) \cdot 3\ (x-1)^2}{(x-1)^6} =$

$= \dfrac{(x-1)^2\ (x - 1 - 3x - 3)}{(x-1)^6} = \dfrac{-2x-4}{(x-1)^4}$

$f'(x) = \dfrac{1}{\frac{x-1}{x+4}} \cdot \dfrac{5}{(x+4)^2} = \dfrac{(x+4)}{(x-1)} \cdot \dfrac{5}{(x+4)^2} =$

$= \dfrac{5}{(x-1)\ (x+4)} = \dfrac{5}{x^2 + 3x - 4}$

$f'(x) = 2\ cos\ (3x - 2) \cdot (-sen\ (3x - 2)) \cdot 3 =$

$= -6\ cos\ (3x - 2)\ sen\ (3x - 2)$

$f'(x) = \dfrac{cos\ x}{2\sqrt{sen\ x}}$

$f'(x) = \dfrac{2x\ cos\ x^2}{sen\ x^2}$

$f'(x) = e^{4x-1} \cdot 4\ sen(3x^2) + e^{4x-1} \cdot cos(3x^2)\ 6x =$

$= 4\ e^{4x-1}\ sen\ (3x^2) + 6x\ e^{4x-1}\ cos\ (3x^2)$

$f'(x) = 2^{4x^2-1} \cdot ln\ 2 \cdot 8x \cdot ln\ (8x) + 2^{4x^2-1} \cdot \dfrac{8}{8x} =$

$= 2^{4x^2-1} \cdot ln\ 2 \cdot 8x \cdot ln\ (8x) + \dfrac{2^{4x^2-1}}{x}$

$f'(x) = \dfrac{2\ (2x+3) \cdot 2\ (1-x) - (2x+3)^2 \cdot (-1)}{(1-x)^2} =$

$= \dfrac{(2x+3)\ (-2x+7)}{(1-x)^2} = \dfrac{-4x^2 + 8x + 21}{(1-x)^2}$

$f'(x) = \left[1 + tg^2\left(\dfrac{2}{x-3}\right)\right] \cdot \dfrac{-2}{(x-3)^2}$

$f'(x) = \dfrac{e^{5x+1} \cdot 5 \cdot (x+2) - e^{5x+1} \cdot 1}{(x+2)^2} =$

$= \dfrac{e^{5x+1}\ (5x+9)}{(x+2)^2}$

$f'(x) = \dfrac{2\ ln\ x \cdot \frac{1}{x} \cdot x - ln^2\ x \cdot 1}{x^2} = \dfrac{2\ ln\ x - ln^2\ x}{x^2}$

$f'(x) = \dfrac{(e^x + x\ e^x)\ (x+2) - x\ e^x}{(x+2)^2} =$

$= \dfrac{(x^2 + 2x + 2)\ e^x}{(x+2)^2}$

$f'(x) = \dfrac{\frac{1}{2\sqrt{x-1}} \cdot (3x+4) - \sqrt{x-1} \cdot 3}{(3x+4)^2} =$

$= \dfrac{-3x+2}{2\sqrt{x-1}\ (3x+4)^2}$

$f'(x) = \dfrac{1}{2\sqrt{\frac{3x+1}{x+2}}} \cdot \dfrac{3\ (x+2) - (3x+1) \cdot 1}{(x+2)^2} =$

$= \dfrac{\sqrt{x+2}}{2\sqrt{3x+1}} \cdot \dfrac{5}{(x+2)^2}$

$f'(x) = \dfrac{2x}{1 + (x^2+2)^2} = \dfrac{2x}{x^4 + 4x^2 + 5}$

$f'(x) = \dfrac{1}{2\sqrt{arc\ tg\ x}} \cdot \dfrac{1}{1+x^2} = \dfrac{1}{2(1+x^2)\sqrt{arc\ tg\ x}}$

$f'(x) = \dfrac{3}{4} \cdot \dfrac{2}{\sqrt{1 - (2x-1)^2}} = \dfrac{3}{2\sqrt{-4x^2 + 4x}}$

$f'(x) = \dfrac{-1}{\sqrt{1 - (\sqrt{x})^2}} \cdot \dfrac{1}{2\sqrt{x}} = \dfrac{-1}{\sqrt{1-x}} \cdot \dfrac{1}{2\sqrt{x}} =$

$= \dfrac{-1}{2\sqrt{x - x^2}}$

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1. $f'(x) = \dfrac{-x^2 + 4x - 10}{(x-2)^2}$, $f'(1) = \dfrac{-15}{9}$;

   $f'(3) = -7$; $f'(5) = \dfrac{-15}{9}$

2. $f'(x) = \dfrac{4}{3}\left(\dfrac{x}{3} + 1\right)^3$, $f'(-4) = \dfrac{-4}{81}$; $f'(-3) = 0$;

   $f'(0) = \dfrac{4}{3}$; $f'(1) = \dfrac{256}{81}$

3. $f'(x) = \dfrac{e^x - e^{-x}}{2}$, $f'(-2) \simeq 3{,}63$; $f'(0) = 0$;

   $f'(3) = 10{,}02$