Identidades trigonométricas

54) a) $\dfrac{\operatorname{sen} 4\alpha + \operatorname{sen} 2\alpha}{\cos 4\alpha + \cos 2\alpha} =$

$$\frac{\cancel{2} \operatorname{sen} \frac{4\alpha+2\alpha}{2} \cdot \cancel{\cos \frac{4\alpha-2\alpha}{2}}}{\cancel{2} \cos \frac{4\alpha+2\alpha}{2} \cdot \cancel{\cos \frac{4\alpha-2\alpha}{2}}} = \frac{\operatorname{sen} 3\alpha}{\cos 3\alpha} = \underline{tg\, 3\alpha}$$

b) $\dfrac{\operatorname{sen} 2\alpha}{1-\cos^2\alpha} = \dfrac{2\cancel{\operatorname{sen}\alpha}\cos\alpha}{\operatorname{sen}^{\cancel{2}}\alpha} = \dfrac{2\cos\alpha}{\operatorname{sen}\alpha} = \underline{2\cot g\,\alpha}$

c) $\dfrac{2\cos(45+\alpha)\cdot\cos(45-\alpha)}{\cos 2\alpha} =$

$$\left(45+\alpha = \frac{A+B}{2} \Rightarrow 45+\alpha = \frac{A}{2} + \frac{B}{2} \Rightarrow A = 2.45^\circ \ y\ B = 2\alpha \right)$$

$$= \frac{\cos 90^\circ + \cos 2\alpha}{\cos 2\alpha} = \underline{1}$$

d) $2tg\,x \cos^2 \frac{x}{2} - \operatorname{sen} x = \cancel{2} tg\,x \cdot \left(\dfrac{1+\cos x}{\cancel{2}}\right) - \operatorname{sen} x =$

$$= \frac{\operatorname{sen} x}{\cos x} + \frac{\operatorname{sen} x}{\cancel{\cos x}} \cdot \cancel{\cos x} - \operatorname{sen} x = \underline{tg\, x}$$

e) $2\operatorname{tg}\alpha \cdot \operatorname{sen}^2\frac{\alpha}{2} + \operatorname{sen}\alpha = 2\operatorname{tg}\alpha \cdot \left(\frac{1-\cos\alpha}{\cancel{2}}\right) + \operatorname{sen}\alpha$

$\frac{\operatorname{sen}\alpha}{\cos\alpha} - \frac{\operatorname{sen}\alpha}{\cancel{\cos\alpha}} \cdot \cancel{\cos\alpha} + \operatorname{sen}\alpha = \operatorname{tg}\alpha$

f) $\frac{\cos(a+b)+\cos(a-b)}{\operatorname{sen}(a+b)+\operatorname{sen}(a-b)} = \frac{\cos a.\cos b - \cancel{\operatorname{sen} a.\operatorname{sen} b} + \cos a\cos b + \cancel{\operatorname{sen} a\operatorname{sen} b}}{\operatorname{sen} a.\cos b + \cancel{\operatorname{sen} b.\cos a} + \operatorname{sen} a.\cos b - \operatorname{sen} b.\cos a}$

$\frac{\cancel{2}\cos a\cancel{\cos b}}{\cancel{2}\operatorname{sen} a\cancel{\cos b}} = \operatorname{cotg} a$

55) a) $\frac{1-\cos 2\alpha}{\operatorname{sen}^2\alpha + \cos 2\alpha} = 2\operatorname{tg}^2\alpha$

$\frac{1-(\cos^2\alpha - \operatorname{sen}^2\alpha)}{\operatorname{sen}^2\alpha + (\cos^2\alpha - \operatorname{sen}^2\alpha)} = \frac{1-\cos^2\alpha + \operatorname{sen}^2\alpha}{\cancel{\operatorname{sen}^2\alpha} + \cos^2\alpha - \cancel{\operatorname{sen}^2\alpha}} =$

$= \frac{\operatorname{sen}^2\alpha + \operatorname{sen}^2\alpha}{\cos^2\alpha} = \frac{2\operatorname{sen}^2\alpha}{\cos^2\alpha} = 2\operatorname{tg}^2\alpha$

b) $\operatorname{sen}2\alpha\cos\alpha - \operatorname{sen}\alpha\cos 2\alpha = \operatorname{sen}\alpha$

$2\operatorname{sen}\alpha\cos\alpha.\cos\alpha - \operatorname{sen}\alpha(\cos^2\alpha - \operatorname{sen}^2\alpha) =$

$2\operatorname{sen}\alpha.\cos^2\alpha - \operatorname{sen}\alpha\cos^2\alpha + \operatorname{sen}^3\alpha =$

$\operatorname{sen}\alpha\cos^2\alpha + \operatorname{sen}^3\alpha = \operatorname{sen}\alpha(\cos^2\alpha + \operatorname{sen}^2\alpha) = \operatorname{sen}\alpha$

c) $\cos\alpha \cdot \cos(\alpha-\beta) + \operatorname{sen}\alpha \cdot \operatorname{sen}(\alpha-\beta) = \cos\beta$

$$\cos\alpha \cdot (\cos\alpha \cdot \cos\beta + \operatorname{sen}\alpha \cdot \operatorname{sen}\beta) + \operatorname{sen}\alpha \cdot (\operatorname{sen}\alpha \cos\beta - \cos\alpha \operatorname{sen}\beta)$$

$$\cos^2\alpha \cos\beta + \cancel{\cos\alpha \operatorname{sen}\alpha \operatorname{sen}\beta} + \operatorname{sen}^2\alpha \cos\beta - \cancel{\operatorname{sen}\alpha \operatorname{sen}\beta \cdot \cos\alpha} =$$

$$= \cos^2\alpha \cos\beta + \operatorname{sen}^2\alpha \cdot \cos\beta = \cos\beta(\cos^2\alpha + \operatorname{sen}^2\alpha) = \cos\beta$$

d) $\operatorname{sen}\alpha + \cos\alpha = \sqrt{2}\cos\left(\frac{\pi}{4} - x\right)$

$$\sqrt{2}\cos\left(\frac{\pi}{4} - \alpha\right) = \sqrt{2}\left(\cos\frac{\pi}{4} \cdot \cos\alpha + \operatorname{sen}\frac{\pi}{4} \cdot \operatorname{sen}\alpha\right)$$

$$\sqrt{2}\left(\frac{\sqrt{2}}{2}\cos\alpha + \frac{\sqrt{2}}{2}\operatorname{sen}\alpha\right) = \cos\alpha + \operatorname{sen}\alpha$$

e) $\sec^2 A - tg^2 A = 1$

$$\frac{1}{\cos^2 A} - \frac{\operatorname{sen}^2 A}{\cos^2 A} = \frac{1 - \operatorname{sen}^2 A}{\cos^2 A} = \frac{\cos^2 A}{\cos^2 A} = 1$$

f) $tg\,\frac{A}{2} = \frac{\operatorname{sen} A}{1+\cos A} = \frac{1-\cos A}{\operatorname{sen} A} = \operatorname{cosec} A - \operatorname{cotg} A$

$$tg\,\frac{A}{2} = \sqrt{\frac{1-\cos A}{1+\cos A}} = \sqrt{\frac{(1-\cos A)(1+\cos A)}{(1+\cos A)(1+\cos A)}} = \sqrt{\frac{1-\cos^2 A}{(1+\cos A)^2}} = \frac{\operatorname{sen} A}{1+\cos A}$$

$$\frac{\operatorname{sen} A}{1+\cos A} = \frac{\operatorname{sen} A}{1+\cos A} \cdot \frac{\operatorname{sen} A}{\operatorname{sen} A} = \frac{\operatorname{sen}^2 A}{(1+\cos A) \cdot \operatorname{sen} A} = \frac{1-\cos^2 A}{(1+\cos A) \cdot \operatorname{sen} A} = \frac{(1-\cos A)(1+\cos A)}{(1+\cos A) \cdot \operatorname{sen} A}$$

$$= \frac{(1-\cos A)}{\operatorname{sen} A}$$

$$\frac{1-\cos A}{\operatorname{sen} A} = \frac{1}{\operatorname{sen} A} - \frac{\cos A}{\operatorname{sen} A} = \operatorname{cosec} A - \operatorname{cotg} A$$

g) $\dfrac{2\,\text{sen}\,\alpha - \text{sen}\,2\alpha}{2\,\text{sen}\,\alpha + \text{sen}\,2\alpha} = \dfrac{1-\cos\alpha}{1+\cos\alpha} = \text{tg}^2\,\dfrac{\alpha}{2}$

$$\frac{2\,\text{sen}\,\alpha - 2\,\text{sen}\,\alpha\cos\alpha}{2\,\text{sen}\,\alpha + 2\,\text{sen}\,\alpha\cos\alpha} = \frac{2\,\text{sen}\,\alpha\,(1-\cos\alpha)}{2\,\text{sen}\,\alpha\,(1+\cos\alpha)} = \frac{1-\cos\alpha}{1+\cos\alpha}$$

$$\frac{1-\cos\alpha}{1+\cos\alpha} = \text{tg}^2\,\frac{\alpha}{2}$$

$$\text{tg}\,\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

h) $\text{sen}^2\,\dfrac{\alpha+\beta}{2} - \text{sen}^2\,\dfrac{\alpha-\beta}{2} = \text{sen}\,\alpha\,.\,\text{sen}\,\beta$

$$\text{sen}^2\,\frac{\alpha+\beta}{2} - \text{sen}^2\,\frac{\alpha-\beta}{2} = \left(\text{sen}\,\frac{\alpha+\beta}{2} + \text{sen}\,\frac{\alpha-\beta}{2}\right)\cdot\left(\text{sen}\,\frac{\alpha+\beta}{2} - \text{sen}\,\frac{\alpha-\beta}{2}\right) =$$

$$= \left(2\,.\,\text{sen}\,\frac{\frac{\alpha+\beta}{2}+\frac{\alpha-\beta}{2}}{2}\,.\,\cos\,\frac{\frac{\alpha+\beta}{2}-\frac{\alpha-\beta}{2}}{2}\right)\cdot\left(2\cos\,\frac{\frac{\alpha+\beta}{2}+\frac{\alpha-\beta}{2}}{2}\,.\,\text{sen}\,\frac{\frac{\alpha+\beta}{2}-\frac{\alpha-\beta}{2}}{2}\right) =$$

$$= \left(2\,.\,\text{sen}\,\frac{\frac{\cancel{2}\alpha}{\cancel{2}}}{2}\,.\,\cos\,\frac{\frac{\cancel{2}\beta}{\cancel{2}}}{2}\right)\cdot\left(2\,.\,\cos\,\frac{\frac{\cancel{2}\alpha}{\cancel{2}}}{2}\,.\,\text{sen}\,\frac{\frac{\cancel{2}\beta}{\cancel{2}}}{2}\right) =$$

$$= \left(2\,\text{sen}\,\frac{\alpha}{2}\,.\cos\,\frac{\beta}{2}\right)\cdot\left(2\cos\,\frac{\alpha}{2}\,.\,\text{sen}\,\frac{\beta}{2}\right) = \text{sen}\,\alpha\,.\,\text{sen}\,\beta$$

i) $\operatorname{sen}^2 A = \frac{1}{2} - \frac{1}{2}\cos 2A$

$$\frac{1}{2} - \frac{1}{2}\cos 2A = \frac{1}{2} - \frac{1}{2}(\cos^2 A - \operatorname{sen}^2 A) =$$

$$= \frac{1}{2} - \frac{1}{2}\cos^2 A + \frac{1}{2}\operatorname{sen}^2 A = \frac{1}{2} - \frac{1}{2}(1-\operatorname{sen}^2 A) + \frac{1}{2}\operatorname{sen}^2 A =$$

$$= \frac{1}{2} - \frac{1}{2} + \frac{1}{2}\operatorname{sen}^2 A + \frac{1}{2}\operatorname{sen}^2 A = \operatorname{sen}^2 A$$

j) $\dfrac{2\operatorname{sen} x}{\operatorname{tg} 2x} = \cos x - \operatorname{sen} x \operatorname{tg} x$

$$\frac{2\operatorname{sen} x}{\dfrac{2\operatorname{tg} x}{1-\operatorname{tg}^2 x}} = \frac{\cancel{2}\operatorname{sen} x(1-\operatorname{tg}^2 x)}{\cancel{2}\operatorname{tg} x} = \frac{\operatorname{sen} x(1-\operatorname{tg}^2 x)}{\dfrac{\operatorname{sen} x}{\cos x}} =$$

$$= \cos x(1-\operatorname{tg}^2 x) = \cos x - \cos x \cdot \frac{\operatorname{sen}^2 x}{\cos^2 x} =$$

$$= \cos x - \operatorname{sen} x \cdot \operatorname{tg} x$$

k) $\dfrac{2\operatorname{tg}\frac{x}{2}}{1+\operatorname{tg}^2\frac{x}{2}} = \operatorname{sen} x$

$$\frac{2\cdot\sqrt{\dfrac{1-\cos x}{1+\cos x}}}{1+\dfrac{1-\cos x}{1+\cos x}} = \frac{2\cdot\sqrt{\dfrac{(1-\cos x)^2}{1-\cos^2 x}}}{\dfrac{1+\cancel{\cos x}+1-\cancel{\cos x}}{1+\cos x}} = \frac{\dfrac{2\cdot(1-\cos x)}{\operatorname{sen} x}}{\dfrac{2}{1+\cos x}} =$$

$$\frac{(1-\cos x)(1+\cos x)}{\operatorname{sen} x} = \frac{1-\cos^2 x}{\operatorname{sen} x} = \operatorname{sen} x$$

l) $$\sqrt{\frac{1+\operatorname{sen} x}{1-\operatorname{sen} x}} = \sec x + \operatorname{tg} x$$

$$\sqrt{\frac{(1+\operatorname{sen} x)^2}{1-\operatorname{sen}^2 x}} = \frac{1+\operatorname{sen} x}{\cos x} = \sec x + \operatorname{tg} x$$

m) $$\frac{\cos 2x - 1}{\cos 2x + 1} = -\operatorname{tg}^2 x$$

$$\frac{\cos 2x - 1}{\cos 2x + 1} = \frac{\cos^2 x - \operatorname{sen}^2 x - 1}{\cos^2 x - \operatorname{sen}^2 x + 1} = \frac{-\operatorname{sen}^2 x - \operatorname{sen}^2 x}{\cos^2 x + \cos^2 x} =$$

$$= \frac{-2\operatorname{sen}^2 x}{2\cos^2 x} = -\operatorname{tg}^2 x$$